RADIATIVE-CONVECTIVE HEAT TRANSFER IN A PLANE LAYER OF A SELECTIVELY ABSORBING MIST

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This paper deals with the thermal field in a plane layer of selectively absorbing gas which has been injected into a steady turbulent stream of high-temperature gas flowing around a porous plate. The boundary-value problem in terms of the energy equation reduces to a nonlinear integral equation in terms of a dimensionless temperature, and this equation is solved numerically by the Newton–Kantorovich method. The results are presented on graphs of temperature and thermal flux in the absorbing gas layer as functions of the space coordinate. Such a problem has been analyzed in [1] for the case of an injected gray gas.

In a plane layer of a selectively absorbing medium the heat is transmitted by radiation, convection, and molecular heat conduction. The physical properties of the medium and the optical properties of its boundary surfaces are assumed constant. The velocity of the injected gas is a certain function of the space coordinate. The formulation of the problem is based on a one-dimensional heat flow.

For this particular case the one-dimensional energy equation and the boundary conditions at the respective surfaces of a plane layer are written as

$$\rho c_p w(y)\,\frac{dT}{dy} = \lambda\,\frac{d^2T}{dy^2} - \frac{dE}{dy},\quad 0 \leqslant y \leqslant \delta \tag{1}$$

$$T(0) = T_1,\quad T(\delta) = T_2 \tag{2}$$

Here E is the resultant hemispheric radiation density and w is the velocity of the injected gas. The meaning of other symbols is as usual.

The boundary-value problem (1), (2) is converted to dimensionless form:

$$\frac{d^2\theta}{d\xi^2} = S_k\left[\frac{d\varphi}{d\xi} + B_0 f(\xi)\,\frac{d\theta}{d\xi}\right],\quad 0 \leqslant \xi \leqslant 1 \tag{3}$$

$$\theta(0) = \theta_1,\quad \theta(1) = \theta_2 \tag{4}$$

$$\left(\theta = \frac{T}{T^*},\quad \theta_i = \frac{T_i}{T_*},\quad \varphi = \frac{E}{\sigma_0 T_*^4},\quad S_k = \frac{\sigma_0 T_*^3\delta}{\lambda}\quad B_0 = \frac{\rho c_p w}{\sigma_0 T_*^3},\quad \xi = \frac{y}{\delta},\quad f(\xi) = \frac{w(\xi)}{w_*}\right)$$

Here the dimensionless quantities are the gas stream velocity f, the gas temperature θ, the net thermal flux density Φ, the parameters B_0 and S_k characterizing the radiation-to-convection and the radiation-to-conduction ratios in the total thermal flux, while the dimensional reference quantities are the characteristic temperature T_* and the gas stream velocity w_*; σ_0 is the Stefan–Boltzmann constant and δ is the thickness of the plane layer.

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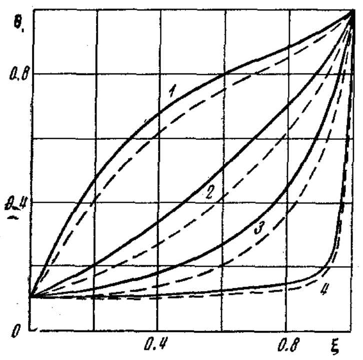

Fig. 1

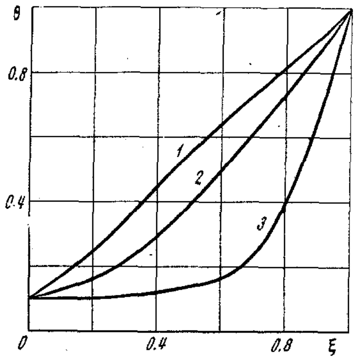

Fig. 2

The divergence of the radiant flux is defined by the following integral relation [2]:

$$\frac{d\Phi}{d\xi} = D\int_{z_1}^{z_2} h_z\left\{2\varepsilon_z(\xi) - w_{1z}(\xi) - \int_0^1 w_{2z}(\xi, x)\,\varepsilon_z(x)\,dx\right\}dz \tag{5}$$

$$\varepsilon_z(\xi) = \frac{z^3}{[\exp(\omega z/\theta(\xi)) - 1]}$$

$$w_{1z}(\xi) = \alpha_z\{[a_1\varepsilon_{1z} + 2a_2r_1\varepsilon_{2z}K_3(h_z)]\,K_2(\tau) + [a_2\varepsilon_{2z} + 2a_1r_2\varepsilon_{1z}K_3(h_z)]\,K_2(h_z - \tau)\}$$

$$w_{2z}(\xi, x) = h_z\{K_1|\tau - t| + 2\alpha_z[r_1(K_2(t) + 2r_2K_3(h_z)\,K_2(h_z - t)) \times$$
$$\times K_2(\tau) + r_2(K_2(h_z - t) + 2r_1K_3(h_z)\,K_2(t))\,K_2(h_z - \tau)]\}$$

$$\left(K_n(\tau) = \int_0^1 \mu^{n-1}\exp(-\tau/\mu)\,d\mu, \quad \tau = h_z\xi, \quad t = h_z x\right.$$

$$\left.\alpha_z = [1 - 4r_1r_2K_3^2(h_z)]^{-1}, \quad z_i = \frac{\nu_i}{\nu_*}, \quad D = 30\left(\frac{\omega}{\pi}\right)^4, \quad \omega = \frac{h\nu_*}{kT_*}\right).$$

Here h_z is the optical layer thickness, a_i and r_i are the absorptivity and the reflectivity of respective layer surfaces, ν and ν_* are the variable and the characteristic spectral frequency of the injected gas, k and h are the Boltzmann and the Plack constants, ε_z and ε_{iz} are the Planck function at the variable temperature θ and at temperature θ_i of the respective layer surfaces, and K_n are integrals of exponential functions ($i = 1, 2;\ n = 1, 2, 3$).

It is evident from (5) that $d\varphi/d\xi$ is an integral expression nonlinear with respect to θ, and this makes Eq. (3) a nonlinear integrodifferential equation. Consequently, a solution to the boundary-value problem (3), (4) cannot be obtained in an analytical form. With the aid of Green's function, Eq. (3) can be reduced to a nonlinear integral equation in θ:

$$\theta(\xi) = [\theta_1\,\mathrm{sh}\,(1 - \xi) + \theta_2\,\mathrm{sh}\,\xi]\left[\mathrm{sh}\,1 + S_k\int_0^1 F(\theta)\,G(\xi, x)\,dx\right]^{-1} \tag{6}$$

$$F(\theta) = \frac{d\Phi}{dx}G(\xi, x) - B_0\frac{d}{dx}[f(x)\,G(\xi, x)]$$

$$G(\xi, x) = \begin{cases} -\dfrac{\mathrm{sh}\,x\,\mathrm{sh}\,(1 - \xi)}{\mathrm{sh}\,1} & (x \leqslant \xi) \\[2ex] -\dfrac{\mathrm{sh}\,\xi\,\mathrm{sh}\,(1 - x)}{\mathrm{sh}\,1} & (x \geqslant \xi) \end{cases}$$

Here G is Green's function of the modified linear part of the differential operator in (3).

The reduction of the integrodifferential equation (3) to the integral equation (6) makes it possible to solve the problem numerically by means of efficient iteration procedures.

For solving Eq. (6) numerically, the integral is approximated by a Gauss quadrature. After that, it reduces further to a system of nonlinear algebraic equations the order of which depends on the number of Gauss points. The resulting system of equations is solved by the Newton–Kantorovich iteration method [3].

The solution of (6) for various combinations of parameter values characterizing the optical properties of the porous plate, the rate of selectively absorbing gas injection, and the radiation-to-convection

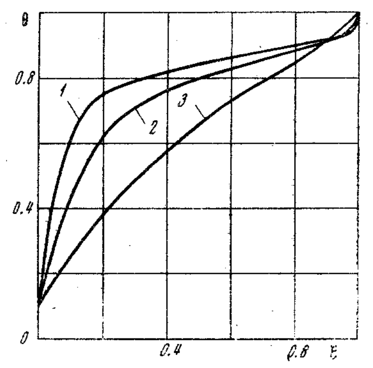

Fig. 3

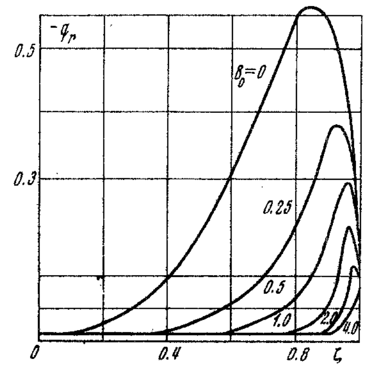

Fig. 4

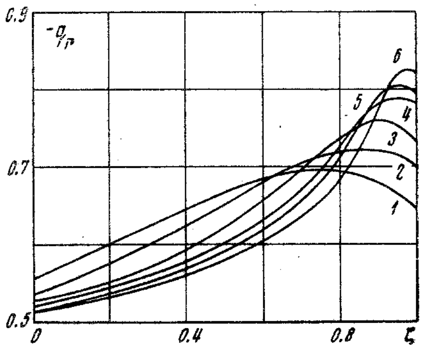

Fig. 5

ratio in the total heat-transfer process is shown in the form of temperature curves with $\theta_1 = 0.1$ and $\theta_2 = 1.0$. The injected medium here is gaseous carbon dioxide, whose absorption spectrum has been taken from [4]. S_k is used as the parameter characterizing the radiation-to-conduction ratio. The (dimensionless) temperature distribution across a layer is shown in Fig. 1 for $S_k = 10$ and $f(\xi) = 1$: the solid lines correspond to $r_1 = 0.9$ and $r_2 = 0.1$; the dashed lines correspond to $r_1 = r_2 = 0.5$; curves 1, 2, 3, and 4 correspond to $B_0 = 0$, 0.5, 1.0, and 4.0, respectively.

A large drop of the temperature level is noted in the layer when the injection parameter B_0 increases. Already at $B_0 \geq 4$ the temperature distribution across the layer becomes uniform and the temperature becomes equal to that of the injected gas, up to the hot surface of the layer. Furthermore, the shaping of the temperature profile is affected by the optical properties of the porous plate. As the reflectivity of the porous plate surface increases, the temperature level drops somewhat.

When the velocity profile in the injected gas is linear, $f(\xi) = 1-\xi$, the effect of the injection parameter B_0 on the temperature profile in the layer becomes somewhat weaker. The (dimensionless) temperature distribution across the layer with $\theta_1 = 0.1$ and $\theta_2 = 1.0$ is shown in Fig. 2 for $S_k = 10$, $f(\xi) = 1-\xi$, and $r_1 = r_2 = 0.1$; curves 1, 2, and 3 correspond to $B_0 = 0.5$, 1.0, and 4.0, respectively. The cooled zone is narrower when $B_0 \geq 4$ than when the velocity profile in the injected gas is uniform.

The effect of the radiation-to-conduction ratio S_k on the shaping of temperature profiles at constant values of the injection parameter B_0 is illustrated in Fig. 3. The (dimensionless) temperature distribution across a layer with $\theta_1 = 0.1$ and $\theta_2 = 1.0$ is shown here for $B_0 = 0.15$, $f(\xi) = 1$, $r_1 = 0.9$, and $r_2 = 0.1$; curves 1, 2, and 3 correspond to $S_k = 1000$, 100, and 10, respectively. The observed pattern resembles in many respects the thermal field in an absorbing layer which is molecularly heat conductive only.

It is to be noted that, during injection of a selectively absorbing gas, the temperature field remains qualitatively the same as when a gray gas is injected [1]. From the standpoint of thermal protection, however, a mist of selectively absorbing gas may be more effective (the cooled zone in a layer widens) than a mist of gray gas. Of practical interest is the calculation of the total thermal flux and its individual components (conductive, convective, and radiative). A single formal integration of Eq. (3) yields a formula for the total thermal flux, when the gas velocity is uniform across the layer thickness:

$$q = q_T + q_k + \varphi = \text{const} \tag{7}$$

$$q_T = -d\theta/d\xi, \qquad q_k = B_0\theta(\xi)S_k, \qquad q_r = \varphi$$

$$\varphi(\xi) = \frac{a_1a_2}{1-r_1r_2}(\theta_1^4 - \theta_2^4) + \int\limits_{z_1}^{z_2}\left[\varphi_z(h_z\xi) - \frac{D}{2}\frac{a_1a_2}{1-r_1r_2}(\varepsilon_{1z} - \varepsilon_{2z})\,dz\right]$$

$$\varphi_z(h_z\xi) = h_z\left\{2\varepsilon_z(\xi) - w_{1z}(\xi) - \int\limits_0^1 w_{2z}(\xi, x)\,\varepsilon_z(x)\,dx\right\}S_k$$

Here q_T, q_k, and φ are the fractional values of the conductive, the convective, and the radiative components, respectively.

The calculation of the total thermal flux and its components presents no difficulties, since the needed values of temperature θ have been obtained earlier from the solution to the integral equation (6). Calculations were made for $S_k = 10$ and $f(\xi) = 1$ with $\theta_1 = 0.1$ and $\theta_2 = 1.0$. The radiative component of the thermal flux $\varphi(\xi)$ as a function of the dimensionless space coordinate has been plotted in Fig. 4 for various values of B_0 with $r_1 = 0.9$ and $r_2 = 0.1$.

Analogous graphs for the radiative component are shown in Fig. 5 for $S_k = 10, f(\xi) = 1$, and $r_1 = r_2 = 0.1$; curves 1, 2, 3, 4, 5, and 6 correspond to $B_0 = 0, 0.25, 0.5, 1.0, 2.0$, and 4.0, respectively. The characteristic maxima are noted here (in Fig. 4 and Fig. 5) to be shifting toward the hot surface as the B_0 is increased. Such a shifting of the maxima can be explained by the effect of the convective component on the total thermal flux and, particularly, on the radiative component. The maxima of the thermal flux are determined largely by the optical properties of the hot surface. When the emissivity is low (Fig. 4), then the maximum radiative flux decreases as the B_0 number is increased and, vice versa, the trend of the relation between this thermal flux and the B_0 number reverses when the hot surface is a good insulator (Fig. 5).

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